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# Guided waves attenuation in water immersed corrugated plates

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## Abstract

Influences of surface corrugations on the propagation of guided waves along an immersed elastic plate are investigated. The Finite Elements Method is used to compute the reflected and transmitted pressure fields for oblique incident plane harmonic waves in a selected frequency range. The effects of corrugations can also be accounted by means of a rheological model. The corrugated surface is then modeled by using modified boundary conditions at the liquid – corrugated plate interface. In this condition a parameter is introduced that can be evaluated by a fit procedure between the analytical solutions of modal resonance peaks and the FEM results for the corrugated plate.

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**Keywords:** ultrasonic interface characterization ; scattering theory ; rough surface;modal resonance ; rheological model;corrugated plate

## 1. Introduction

In many domains such as the study of corroded immersed surfaces we use guided Lamb waves. Usually rough interfaces correspond to the case of small roughness. It is shown that the real part of the wave vector is only slightly affected, while the imaginary part is substantially affected. A description of a rough interface using a rheological model has already been mentioned [1]. The rough interface is replaced by a flat one and the boundary

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conditions on the liquid-solid interface are adapted. This model consists in associating with the interface, a surface distribution of springs [2].

In this work, we replace the roughness by a simple geometrical pattern and we use a computer code based on the finite element method to simulate the acoustic scattering by the plate and get the transmission coefficient. The characteristics of the resonances exhibited in the transmission coefficient are obtained within the classical model of resonances [3]. Then we calculate the characteristics of Lamb waves by using the rheological model. The resonances are obtained from the complex zeroes of the characteristic equation. We deduce the characteristics of the Lamb waves by replacing the rough interface by a rheological model.

## 2. Simulation of the acoustic scattering by a rough plate

We consider a plate with a simple geometrical periodic pattern immersed in water considered as a perfect fluid. The height  $h$  (Fig.1) will be considered as a variable parameter, but it will always remain small compare to the thickness of the plate. The width of the basic cell is comparable to the wavelength of the Lamb wave. We consider in all this work a plane strain two-dimensional problem. The plate is made of stainless steel of density  $\rho_s=7800 \text{ kg/m}^3$ , of longitudinal/transversal wave speed  $C_{L/T}=6020 / 3220 \text{ m/s}$ . The water density is  $1000 \text{ kg/m}^3$  and the sound speed in the water is  $c_w = 1500 \text{ m/s}$ .

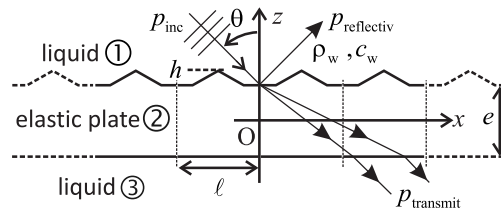


Fig. 1 Geometry of the immersed plate

The numerical study was carried out using a finite element model implemented in commercially software [4]. A harmonic progressive plane wave  $p_{inc} = p_0 e^{i(\vec{k}_{inc} \cdot \vec{r} - \omega t)}$  is sent through the fluid towards the plate under the incidence angle  $\theta$  (Fig. 1). The transmission coefficient  $T$  is calculated at first for a flat plate, to identify the classical Lamb modes [3] (Fig. 2a). The frequency is fixed at 600 kHz. The corresponding curves for the various values of  $h$  are then obtained (for example, the  $A_1$  mode in the Fig.2b).

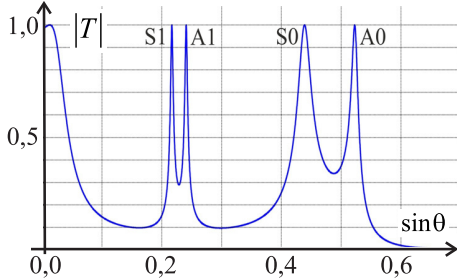


Fig. 2a  $|T|$  versus  $\sin \theta$  ( $h = 0$ )

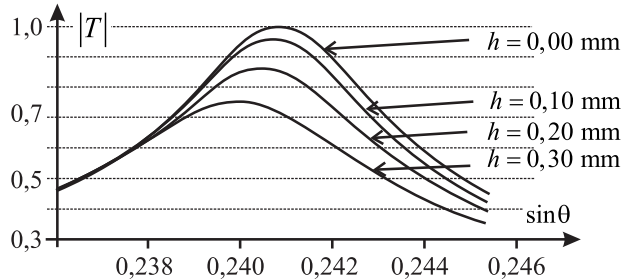


Fig. 2b  $|T|$  versus  $\sin \theta$  for some values of  $h$ , for the  $A_1$  mode.

For each mode, and for different values of  $h$ , about 100 values of  $T$  around the resonances are considered. In the vicinity of a well separated resonance, the transmission coefficient is written as the contribution of this single resonance and the model of Breit-Wigner [3] allows us to write :

$$T \propto (\sin \underline{\theta} - \sin \underline{\theta}_p)^{-1} \quad (1)$$

where  $\sin \underline{\theta}_p$  is the complex pole related to the resonance of the relevant mode. The complex pole can be written as:

$$\sin \underline{\theta}_p = \sin \theta_{\max} + i\gamma / 2 \quad (2)$$

The resonance characteristics are :  $\sin \theta_{\max}$  corresponding to the maximum of the transmission coefficient, and  $\gamma$  its width at  $-3\text{dB}$ . The determination of these quantities is done by the least squares fitting procedure in the vicinity of the maximum. The correlation coefficient between the computed data and the values of  $T$  obtained from Eq. 1 is at least 0,999 for all the identifications. The scattering resonances of the  $A_1$  and  $S_1$  modes have been characterized for each value of the height  $h$ . For each mode, we take the values found for  $\sin \theta_{\max}$  and  $\gamma / 2$  versus  $h$ , and we study the relative values defined by :

$$\eta_\theta = 100 \frac{\sin \theta_{\text{plane plate}} - \sin \theta_{\max}}{\sin \theta_{\text{plane plate}}} \quad \eta_\gamma = 100 \frac{(\gamma / 2)_{\text{plane plate}} - \gamma / 2}{(\gamma / 2)_{\text{plane plate}}} \quad (3a/b)$$

The variations of  $\eta_\theta$  are always lower than 1%. The parameter  $\eta_\gamma$  (fig 3a/b) exhibits variations up to 30% indicating that this parameter is highly sensitive to the roughness.

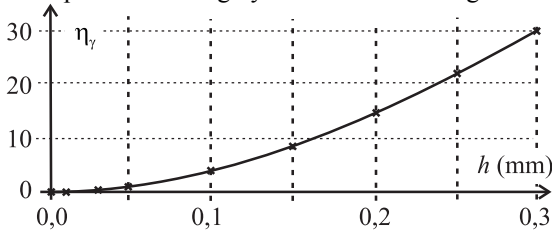


Fig.3a Variations of  $\eta_\theta$  versus  $h$  for the  $A_1$  mode

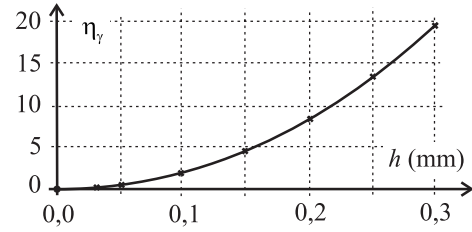


Fig.3b Variations of  $\eta_\gamma$  versus  $h$  for the  $S_1$  mode

### 3. Rheological model

In the frame of the empirical rheological model, the rough interface is replaced by a flat interface with special boundary conditions. In the model of Jones [2, 3], an uniform distribution of longitudinal and shear springs is introduced. Let  $RL$  for the tension in the  $Oz$  direction. In the following, the liquid is perfect, consequently, the boundary conditions at  $+e/2$  (see Fig.1) and  $-e/2$  are written as :

$$\begin{cases} -p_1 = RL(u_{z2} - u_{z1}) \\ \sigma_{zz2} = -p_1 \\ \sigma_{xz2} = 0 \end{cases} \quad \begin{cases} u_{z2} = u_{z3} \\ \sigma_{zz2} = -p_3 \\ \sigma_{xz2} = 0 \end{cases} \quad (4a/b)$$

We are looking for solutions in terms of harmonic waves  $e^{j(\underline{k} \cdot \underline{r} - \omega t)}$  where the angular frequency  $\omega$  is real and the wave vector  $\underline{k}$  is complex. From the boundary conditions (Eq. 4a/b) we deduce a linear system. Its determinant is depending on  $\omega$  (fixed) and  $\underline{k}_x$ . The zeroes of the determinant of this system provide angular resonances in the complex form :  $\underline{k}_x = k'_x + j k''_x$ . The calculation of the zeroes of this determinant is done by the Newton-Raphson method. The link to the previous section 2 is done by the relationship :

$$\sin \underline{\theta} = \underline{k}_x / k_F \quad (5)$$

where  $k_F = \omega / c_{\text{water}}$  is the wave vector of the ultrasonic wave in water. We found that, for the values of  $RL$  greater than  $10^{14} \text{ Nm}^{-3}$ , the values of the complex zeroes of the determinant are the same as those for a flat immersed plate.

We plot the variations of the real and imaginary part of the zeroes versus  $RL$  for both modes. Curves must be read from higher values of  $RL$  ( $10^{14} \text{ Nm}^{-3}$ ) corresponding to the flat plate to the lower values. The relative variation of the real part is not significant but the variation of the imaginary part is greater as it can be seen in figure 4a/b. For this modes, in the range of  $RL$  (6 to  $1.10^{13} \text{ Nm}^{-3}$ ), the imaginary part of the zeroes decreases noticeably (around 15%).

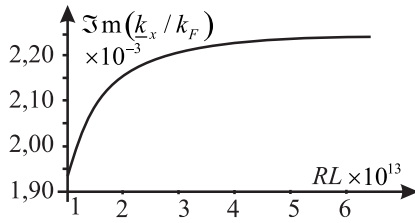


Fig. 4a variations of  $\Im m(k_x / k_F)$  versus  $RL$ ,  $A_1$  mode

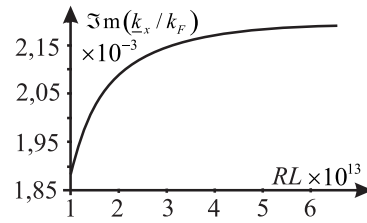


Fig. 4b variations of  $\Im m(k_x / k_F)$  versus  $RL$ ,  $S_1$  mode

#### 4. Results and discussion

The two models, one based on the simulation of a corrugated interface and the other one on the use of an empirical rheological model lead to comparable results. The sensitive parameter is the width of the resonance. In order to get a link between the two approaches, we compare the evolution of the width with  $h$  and, the evolution of the imaginary part of  $k_x$  with  $RL$ . A numerical fit obtained from fig. 4a/b gives for both modes :

$$\gamma / 2 = \alpha_m h^2 + \beta_m \quad (6)$$

where  $\alpha_m, \beta_m$  depend on the concerned mode. In the complex plane, the study of  $\Im m(k_x / k_F)$  versus  $RL$  leads also to a quadratic law (with a correlation coefficient greater than 0,99) for both modes :

$$\Im m(k_x / k_F) = \alpha'_m - \beta'_m (1 / KL)^2 \quad (7)$$

where  $\alpha'_m$  and  $\beta'_m$  depend on the mode. The connection between the two approaches comes from the relation (5) with  $\Im m(\sin \theta) = k_x'' / k_F$ . It has been shown that the value of  $RL$  depends on the frequency. Then relations 5a/b has to be understood as relation between Fourier transforms and the assumption of a real constant  $RL$  could be questionable. In this paper it is considered that the imaginary part of  $RL$  is very small compared to its real part.

#### 5. Conclusion

Results provided by numerical simulations or by a rheological model have been successfully compared for two modes of Lamb and with one parameter  $h$ . Works are in progress by considering  $RL$  as a complex parameter. This new approach could give a better agreement between the numerical experiment and the theoretical approach. Once this is done, it will be possible to predict the effect of the roughness by using a simple rheological model.

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